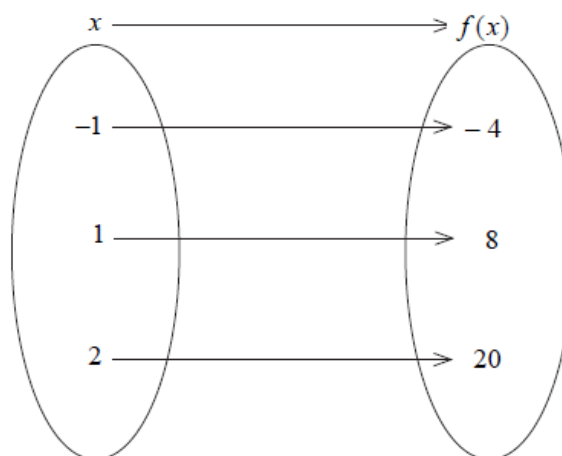


Topic 6 Part 5 [189 marks]

A quadratic function,
 $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



- 1a. Use the mapping diagram to write down **two** equations in terms of a and b . [2 marks]
- 1b. Find the value of a . [1 mark]
- 1c. Find the value of b . [1 mark]
- 1d. Calculate the x -coordinate of the vertex of the graph of $f(x)$. [2 marks]

The function
 $f(x) = 5 - 3(2^{-x})$ is defined for
 $x \geq 0$.

- 2a. On the axes below sketch the graph of $f(x)$ and show the behaviour of the curve as x increases. [3 marks]
- 2b. Write down the coordinates of any intercepts with the axes. [1 mark]
-
- 2c. Draw the line $y = 5$ on your sketch. [1 mark]
- 2d. Write down the number of solutions to the equation $f(x) = 5$. [1 mark]

A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

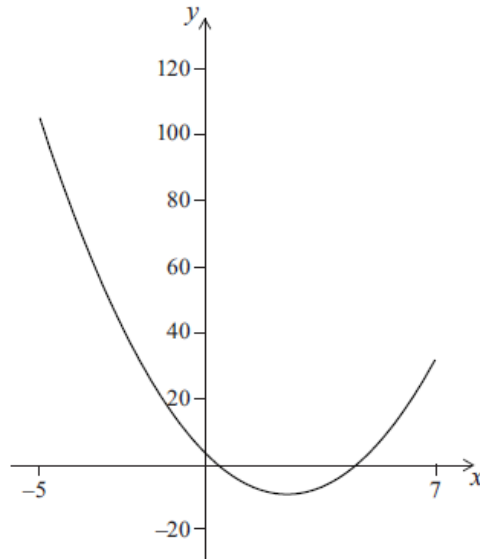
where N is the number of teenagers who have heard the rumour t hours after it is first started.

- 3a. Find the number of teenagers who started the rumour. [2 marks]

3b. Write down the number of teenagers who have heard the rumour five hours after it is first started. [1 mark]

3c. Determine the length of time it would take for 150 teenagers to have heard the rumour. **Give your answer correct to the nearest minute.** [3 marks]

The graph of $y = 2x^2$
 $- rx + q$ is shown for
 $-5 \leq x \leq 7$.



The graph cuts the y axis at (0, 4).

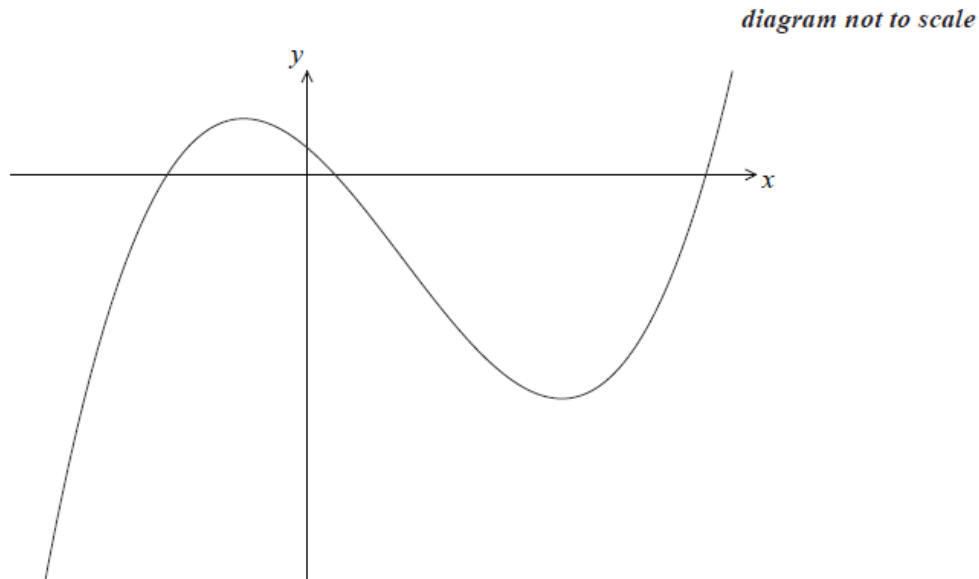
4a. Write down the value of q . [1 mark]

4b. The axis of symmetry is $x = 2.5$. [2 marks]
Find the value of r .

4c. The axis of symmetry is $x = 2.5$. [1 mark]
Write down the minimum value of y .

4d. The axis of symmetry is $x = 2.5$. [2 marks]
Write down the range of y .

The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.



- 5a. Write down the values of x where the graph of $f(x)$ intersects the x -axis. [3 marks]
- 5b. Write down $f'(x)$. [3 marks]
- 5c. Find the value of the local maximum of $y = f(x)$. [4 marks]
- 5d. Let P be the point where the graph of $f(x)$ intersects the y axis. [1 mark]
Write down the coordinates of P.
- 5e. Let P be the point where the graph of $f(x)$ intersects the y axis. [2 marks]
Find the gradient of the curve at P.
- 5f. The line, L , is the tangent to the graph of $f(x)$ at P. [2 marks]
Find the equation of L in the form $y = mx + c$.
- 5g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L . [1 mark]
Write down the gradient of the tangent at Q.
- 5h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L . [3 marks]
Calculate the x -coordinate of Q.

Daniel wants to invest
\$25 000 for a total of three years. There are two investment options.

Option One pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

Option Two pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

- 6a. Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**. [8 marks]
- 6b. Determine Daniel's best investment option. [1 mark]

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \quad n = 1, 2, 3, \dots$$

6c. Calculate u_1 , the first term in the sequence. [2 marks]

6d. Show that the common difference is 7. [2 marks]

6e. S_n is the sum of the first n terms of the sequence. [3 marks]

Find an expression for S_n . Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants.

6f. The first term, v_1 , of a geometric sequence is 20 and its fourth term v_4 is 67.5. [2 marks]

Show that the common ratio, r , of the geometric sequence is 1.5.

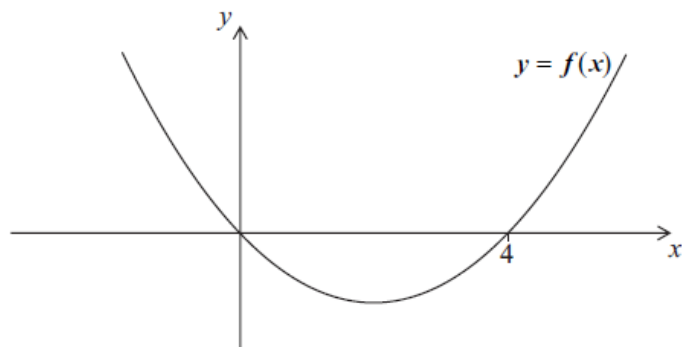
6g. T_n is the sum of the first n terms of the geometric sequence. [2 marks]

Calculate T_7 , the sum of the first seven terms of the geometric sequence.

6h. T_n is the sum of the first n terms of the geometric sequence. [2 marks]

Use your graphic display calculator to find the smallest value of n for which $T_n > S_n$.

The following is the graph of the quadratic function $y = f(x)$.



7a. Write down the solutions to the equation $f(x) = 0$. [2 marks]

7b. Write down the equation of the axis of symmetry of the graph of $f(x)$. [2 marks]

7c. The equation $f(x) = 12$ has two solutions. One of these solutions is $x = 6$. Use the symmetry of the graph to find the other solution. [1 mark]

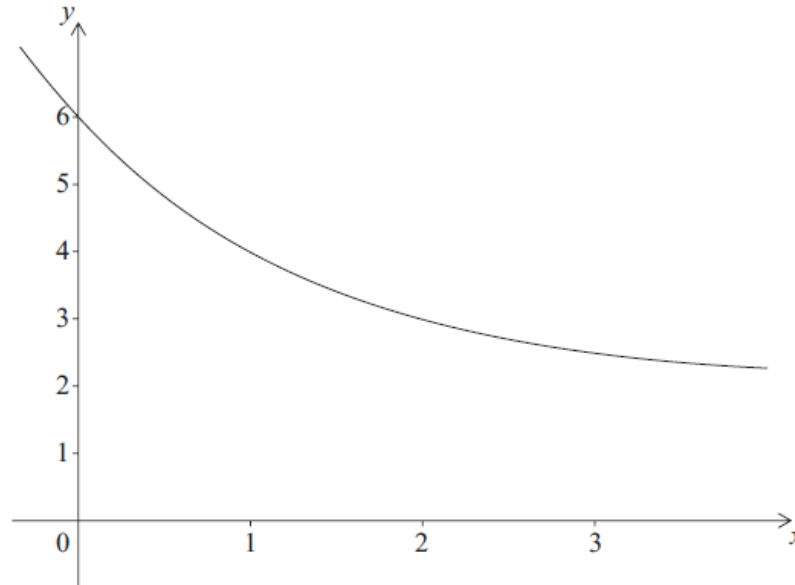
7d. The minimum value for y is -4 . Write down the range of $f(x)$. [1 mark]

Consider the function

$f(x) = p(0.5)^x + q$ where p and q are constants. The graph of $f(x)$ passes through the points

$(0, 6)$ and

$(1, 4)$ and is shown below.



8a. Write down two equations relating p and q . [2 marks]

8b. Find the value of p and of q . [2 marks]

8c. Write down the equation of the horizontal asymptote to the graph of $f(x)$. [2 marks]

Consider the function $f(x) = x^3 - 3x^2 - 24x + 30$.

9a. Write down $f(0)$. [1 mark]

9b. Find $f'(x)$. [3 marks]

9c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. [2 marks]

9d. (i) Use $f'(x)$ to find the x -coordinate of M and of N. [5 marks]
(ii) Hence or otherwise write down the coordinates of M and of N.

9e. Sketch the graph of $f(x)$ for $-5 \leq x \leq 7$ and $-60 \leq y \leq 60$. Mark clearly M and N on your graph. [4 marks]

9f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. [6 marks]

(i) Find the x -coordinate of A and of B.

(ii) Find the y -coordinate of B.

Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.

Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

10a. Find the amount of money that Ruby will have in the bank after 3 years. [2 marks]

10b. Show that Ying will have 7545 USD in the bank at the end of 10 years. [3 marks]

10c. Find the number of complete years it will take for Ying's investment to first exceed 6500 USD. [3 marks]

10d. Find the number of complete years it will take for Ying's investment to exceed Ruby's investment. [3 marks]

10e. Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 Euros. The bank charges 1.8 % commission. [4 marks]

Calculate the amount of money Ruby will invest in the Italian bank after commission.

10f. Ruby returns to the USA for a short holiday. She converts 800 Euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 Euro = 1.29 USD. [5 marks]

Calculate the percentage commission Ruby is charged by the bank.

Given the function

$$f(x) = 2 \times 3^x \text{ for } -2$$

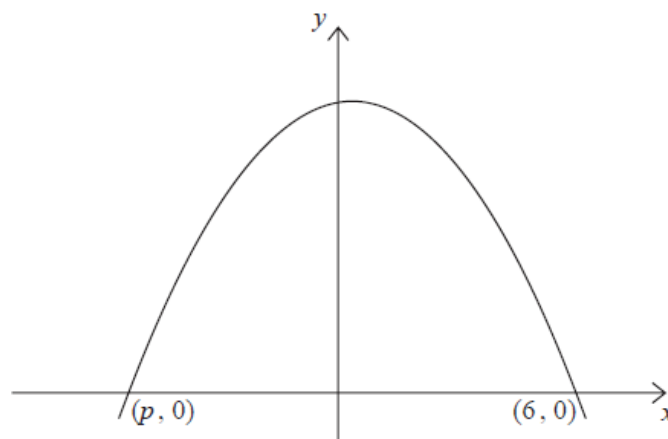
$$\leq x$$

$$\leq 5,$$

11a. find the range of f . [4 marks]

11b. find the value of x given that $f(x) = 162$. [2 marks]

The diagram below shows the graph of a quadratic function. The graph passes through the points (6, 0) and (p, 0). The maximum point has coordinates (0.5, 30.25).



12a. Calculate the value of p . [2 marks]

- 12b. Given that the quadratic function has an equation $y = -x^2 + bx + c$ where $b, c \in \mathbb{Z}$, find b and c . [4 marks]

- 13a. Sketch the graph of $y = 2^x$ for $-2 \leq x \leq 3$. Indicate clearly where the curve intersects the y -axis. [3 marks]

- 13b. Write down the equation of the asymptote of the graph of $y = 2^x$. [2 marks]

- 13c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3 marks]

- 13d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2 marks]

- 13e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. [1 mark]

- 13f. Use Differential Calculus to verify that your answer to (e) is correct. [5 marks]

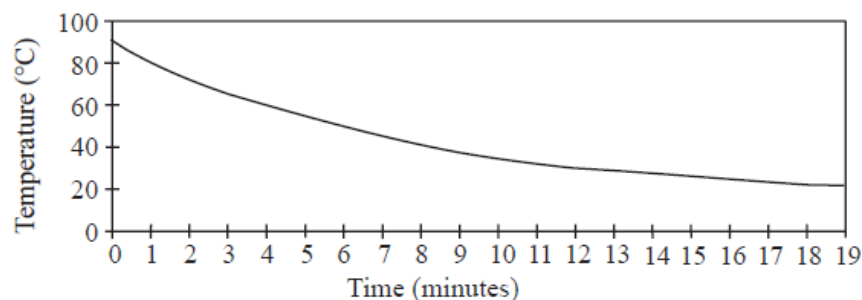
- 13g. The curve $y = px^2 + qx - 4$ passes through the point $(2, -10)$. Use the above information to write down an equation in p and q . [2 marks]

- 13h. The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1. Find $\frac{dy}{dx}$. [2 marks]

- 13i. The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1. Hence, find a second equation in p and q . [1 mark]

- 13j. The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1. Solve the equations to find the value of p and of q . [3 marks]

The following graph shows the temperature in degrees Celsius of Robert's cup of coffee, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the coffee out.



- 14a. Find the initial temperature of the coffee. [1 mark]

14b. Write down the equation of the horizontal asymptote. [1 mark]

14c. Find the room temperature. [1 mark]

14d. Find the temperature of the coffee after 10 minutes. [1 mark]

14e. Find the temperature of Robert's coffee after being heated in the microwave for 30 **seconds** after it has reached the temperature in part (d). [3 marks]

14f. Calculate the length of time it would take a similar cup of coffee, initially at 20°C , to be heated in the microwave to reach 100°C . [4 marks]

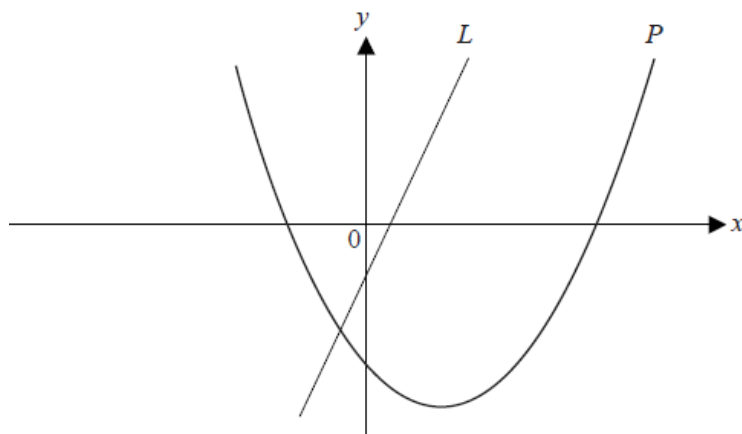
Robert, who lives in the UK, travels to Belgium. The exchange rate is 1.37 euros to one British Pound (GBP) with a commission of 3 GBP, which is subtracted before the exchange takes place. Robert gives the bank 120 GBP.

14g. Calculate **correct to 2 decimal places** the amount of euros he receives. [3 marks]

14h. He buys 1 kilogram of Belgian chocolates at 1.35 euros per 100 g. [3 marks]

Calculate the cost of his chocolates in GBP **correct to 2 decimal places**.

The diagram below shows the graph of a line L passing through $(1, 1)$ and $(2, 3)$ and the graph P of the function $f(x) = x^2 - 3x - 4$



15a. Find the gradient of the line L . [2 marks]

15b. Differentiate $f(x)$. [2 marks]

15c. Find the coordinates of the point where the tangent to P is parallel to the line L . [3 marks]

15d. Find the coordinates of the point where the tangent to P is perpendicular to the line L . [4 marks]

- 15e. Find [3 marks]
- (i) the gradient of the tangent to P at the point with coordinates $(2, -6)$.
 - (ii) the equation of the tangent to P at this point.
- 15f. State the equation of the axis of symmetry of P . [1 mark]
- 15g. Find the coordinates of the vertex of P and state the gradient of the curve at this point. [3 marks]
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